EXERCISE – III   HINTS & SOLUTIONS

Sol.1(i) $T_{r+1} = {}^{11}C_r (ax^2)^{11-r}\ (bx)^{-r} = {}^{11}C_r\, a^{11-r} b^{-r}\ x^{22-3r}$

For $x^7 \Rightarrow 22 - 3r = 7 \Rightarrow 3r = 15 \Rightarrow r = 5$

$\therefore T_6 = T_{5+1} = {}^{11}C_5\, a^6\, b^{-5}\, x^7$

coeff. of $x^7 = {}^{11}C_5 \dfrac{a^6}{b^5}$

(ii) $T_{r+1} = {}^{11}C_r (ax)^{11-r} (-1)^r (bx^2)^{-r}$

$= {}^{11}C_r\, a^{11-r}\, b^{-r} (-1)^r x^{11-3r}$

For $x^{-7} \Rightarrow 11 - 3r = -7 \Rightarrow 18 = 3r \Rightarrow r = 6$

$\therefore T_{6+1} = {}^{11}C_6 \dfrac{a^5}{b^6} x^{-7}$

coeff. of $x^{-7} = {}^{11}C_6 \dfrac{a^5}{b^6}$

(iii) ${}^{11}C_5 \dfrac{a^6}{b^5} = {}^{11}C_6 \dfrac{a^5}{b^6} \Rightarrow a = \dfrac{1}{b} \Rightarrow ab = 1$

Sol.2 In $(1 + x)^{18}$, coeff T_{2r+4} = coef T_{r-2}

$\Rightarrow {}^{18}C_{2r+3} = {}^{18}C_{r-3}$

$\Rightarrow 2r + 3 = r - 3$ or $2r + 3 + r - 3 = 18$

$\Rightarrow r = -6$ or $3r = 18$

$\Rightarrow$ doesn't exist or $r = 6$

Sol.3 In $(1 + x)^{14}$, coeff. r^{th}, $(r+1)^{th}$ $(r+2)^{th}$ in A.P.

$\Rightarrow {}^{14}C_{r-1}, {}^{14}C_r, {}^{14}C_{r+1}$ in A.P.

$\Rightarrow 2 \cdot {}^{14}C_r = {}^{14}C_{r-1} + {}^{14}C_{r+1}$

$$\Rightarrow \frac{2\cdot 14!}{r!(14-r)!} = \frac{14!}{(r-1)!(15-r)!} = \frac{14!}{(r+1)!(13-r)!}$$

$$\Rightarrow \frac{2}{r(14-r)} = \frac{1}{(15-r)(14-r)} + \frac{1}{(r+1)r}$$

$$\Rightarrow \frac{2}{r(14-r)} = \frac{r(r+1)+(15-r)(14-r)}{r(r+1)(14-r)(15-r)}$$

$\Rightarrow 2(r + 1)(15 - r) = r(r + 1) + (15 - r)(14 - r)$

$\Rightarrow 2(15 + 14r - r^2) = r^2 + r + 210 - 29r + r^2$

$\Rightarrow 4r^2 - 56r + 180 = 0 \Rightarrow r^2 - 14r + 45 = 0$

$\Rightarrow (r - 5)(r - 9) = 0 \Rightarrow r = 5$ or 9

Sol.4 (a) $T_{r+1} = {}^{10}C_r \left(\dfrac{x}{3}\right)^{\frac{10-r}{2}} \dfrac{3^{\frac{r}{2}}}{(2x^2)^r} = {}^{10}C_r\, 3^{r-5}\, 2^{-r}\, x^{\frac{10-5r}{2}}$

For constant term $\Rightarrow \dfrac{10-5r}{2} = 0 \Rightarrow r = 2$

$\therefore T_3 = {}^{10}C_2 \dfrac{1}{3^3 2^2} = \dfrac{10\times 9}{2\cdot 3^3\cdot 2^2} = \dfrac{5}{12}$

(b) $T_{r+1} = {}^8C_r \dfrac{x^{\frac{(8-r)}{3}}\, x^{-\frac{r}{5}}}{2^{8-r}} = {}^8C_r \dfrac{1}{2^{8-r}}\, x^{\frac{(40-8r)}{15}}$

For constant term $\Rightarrow \dfrac{40-8r}{15} = 0 \Rightarrow r = 5$

$\therefore T_6 = {}^8C_5 \dfrac{1}{2^3} = \dfrac{8\cdot 7\cdot 6}{3\cdot 2}\dfrac{1}{2^3} = 7$

Sol.5 $$\sum_{r=0}^{n} (-1)^r\, {}^nC_r \left[\frac{1}{2^r} + \frac{3^r}{2^{2r}} + \frac{7^r}{2^{3r}} + \frac{15^r}{2^{4r}} + \ldots\ldots \text{up to m term}\right]$$

$$\sum_{r=0}^{n} (-1)^r\, {}^nC_r \left[\left(1-\frac{1}{2}\right)^r + \left(1-\frac{1}{2^2}\right)^r + \left(1-\frac{1}{2^3}\right)^r + \left(1-\frac{1}{2^4}\right)^r + \ldots\ldots + \left(1-\frac{1}{2^m}\right)^r\right]$$

$$\sum_{r=0}^{n} (-1)^r\, {}^nC_r \left(1-\frac{1}{2}\right)^r + \sum_{r=0}^{n} (-1)^r\, {}^nC_r \left(1-\frac{1}{2^2}\right) + \ldots\ldots + \sum_{r=0}^{n} (-1)^r\, {}^nC_r \left(1-\frac{1}{2^m}\right)^r$$

$$= \left[1-\left(1-\frac{1}{2}\right)\right]^n + \left[1-\left(1-\frac{1}{2^2}\right)\right]^n + \ldots\ldots + \left[1-\left(1-\frac{1}{2^m}\right)\right]^n$$

$$= \left(\frac{1}{2}\right)^n + \left(\frac{1}{2^2}\right)^n + \left(\frac{1}{2^3}\right)^n + \left(\frac{1}{2^4}\right)^n + \ldots + \left(\frac{1}{2^m}\right)^n$$

$$= \frac{1}{2^n} + \frac{1}{2^{2n}} + \frac{1}{2^{3n}} + \frac{1}{2^{4n}} + \ldots\ldots + \frac{1}{2^{mn}}$$

$$= \frac{\frac{1}{2^n}\left[\left(\frac{1}{2^n}\right)^m - 1\right]}{\frac{1}{2^n} - 1} = \frac{\frac{1}{2^n}\left[\frac{1}{2^{nm}} - 1\right]}{\frac{1}{2^n} - 1}$$

$$= \frac{1\ (1 - 2^{mn})}{(1 - 2^n)2^{mn}} = \frac{(2^{mn} - 1)}{(2^n - 1)2^{mn}}$$

Sol.6 In $(1 + x)^{2n}$, coeff of T_2, T_3, T_4 in AP.

i.e. ${}^{2n}C_1$, ${}^{2n}C_2$, ${}^{2n}C_3$ in AP

$\Rightarrow\ 2 \cdot {}^{2n}C_2 = {}^{2n}C_1 + {}^{2n}C_3$

$$\Rightarrow \frac{2 \cdot (2n)!}{2!(2n-2)!} = \frac{(2n)!}{1!(2n-1)!} + \frac{(2n)!}{3!(2n-3)!}$$

$$\Rightarrow \frac{1}{(2n-2)} = \frac{1}{(2n-1)(2n-2)} + \frac{1}{6}$$

$$\Rightarrow \frac{(2n-2)}{(2n-1)(2n-2)} = \frac{1}{6} \Rightarrow 6(n-1) = (2n-1)(n-1)$$

$\Rightarrow\ (2n - 7)(n - 1) = 0 \quad \Rightarrow\quad 2n^2 - 9n + 7 = 0$

Sol.7 $(1 + x + x^2)^n = a_0 + a_1x + a_2x^2 + \ldots\ldots + a_{2n}x^{2n}$

(i) Put x = 1

$3^n = a_0 + a_1 + a_2 + a_3 + \ldots\ldots + a_{2n}$

(ii) Put x = –1

$(1 - 1 + 1)^n = a_0 - a_1 + a_2 + \ldots\ldots + a_{2n}$

$\Rightarrow\ a_0 - a_1 + a_2 - a_3 + \ldots\ldots + a_{2n} = 1$

(iii) Replace $x \longrightarrow -\frac{1}{x}$ in (1)

$$\left(1 - \frac{1}{x} + \frac{1}{x^2}\right)^n = a_0 - a_1\frac{1}{x} + \frac{a_0}{x^2} - \frac{a_3}{x^3} + \ldots + \frac{a_{2n}}{x^{2n}}$$

$$\frac{(x^2 + 1 - x)^n}{x^{2n}} = a_0 - \frac{a_1}{x} + \frac{a_2}{x^2} - \frac{a_3}{x^3} + \ldots + \frac{a_{2n}}{x^{2n}}$$

Multiply equation (1) & (2) and compair coeff. of x

$a_0^2 - a_1^2 + a_2^2 - a_3^2 + \ldots\ldots + a_{2n}^2$

$$= \text{coeff. of } x^{2n} \text{ in } \frac{(1 + x^2 + x)^n(1 + x^2 - x)^n}{x^{2n}}$$

$$= \text{coeff of } x^{2n} \text{ in } \frac{\left[(1 + x^2)^2 - x^2\right]^n}{x^{2n}}$$

$= \text{coeff. of } x^{2n} \text{ in } [1 + x^2 + x^4]^n$

$$= \text{coeff. of } x^{2n} \text{ in } \sum_{P=0}^{2n} a_p x^{2p} = \text{coeff. of } x^{2n} \text{ is } a_n$$

$$\Rightarrow\ a_0^2 - a_1^2 + a_2^2 - a_3^2 + \ldots\ldots + a_{2n}^2 = a_n$$

Sol.8 $(1 + x)^n$ $n \in N$

Let coeff of $(r - 1)^{th}$, r^{th}, $(r + 1)^{th}$ $(r + 2)^{th}$ are given a, b, c, d

${}^nC_{r-2} = a$, ${}^nC_{r-1} = b$, ${}^nC_r = c$, ${}^nC_{r+1} = d$

then $\quad \frac{a}{a+b} + \frac{c}{c+d} = \frac{2b}{b+c}$

$a + b = {}^nC_{r-2} + {}^nC_{r-1} = {}^{n+1}C_{r-1}$

$b + c = {}^nC_{r-1} + {}^nC_r = {}^{n+1}C_r$

$c + d = {}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}$

$$\text{L.H.S.} = \frac{{}^nC_{r-2}}{{}^{n+1}C_{r-1}} + \frac{{}^nC_r}{{}^{n+1}C_{r+1}}$$

$$= \frac{r-1}{n+1} + \frac{r+1}{n+1} = \left(\frac{2r}{n+1}\right)$$

$$\text{R.H.S.} = \frac{2 \cdot {}^nC_{r-1}}{{}^{n+1}C_r}$$

$$= 2 \cdot \frac{n!}{(r-1)!(n-r+1)} \cdot \frac{r!(n-r+1)!}{(n+1)!} = \left(2 \cdot \frac{r}{n+1}\right)$$

$\Rightarrow$ L.H.S. = R.H.S.

Sol.9 $$\left(5^{\frac{2}{5}\log_5\sqrt{4^x+44}} + 5^{\frac{1}{\log_5\sqrt[3]{2^{x-1}+7}}}\right)^8$$

$$= \left(\left(\sqrt{4^x + 44}\right)^{\frac{2}{5}} + \frac{1}{\sqrt[3]{2^{x-1} + 7}}\right)^8$$

$$= \left(\left(4^x + 44\right)^{\frac{1}{5}} + \left(2^{x-1} + 7\right)^{-\frac{1}{3}}\right)^8$$

$$T_4 = {}^8C_3(4^x + 44)^{\frac{8-3}{5}}\ (2^{x-1} + 7)^{-\frac{1}{3}\cdot 3} = 336$$

$\Rightarrow\ {}^8C_3\ (4^x + 44)\ (2^{x-1} + 7)^{-1} = 336$

$\Rightarrow \quad 56 \cdot \dfrac{(4^x + 44)}{(2^{x-1}+7)} = 336 \Rightarrow 4^x + 44 = 6(2^{x-1} + 7)$

$\Rightarrow \quad 2(4^x + 44) = 6(2^x + 14)$

$\Rightarrow (2^x)^2 + 44 = 3(2^x) + 42 \Rightarrow (2^x)^2 - 3(2^x) + 2 = 0$

$\Rightarrow \quad (2^x - 2)(2^x - 1) = 0 \Rightarrow 2^x = 2$ or $2^x = 1$

$\Rightarrow \quad x = 1$ or $x = 0 \Rightarrow x = 0$ or 1

Sol.10 R.H.S. $= {}^nC_{r+1} = {}^{n-1}C_r + {}^{n-1}C_{r+1}$

$= {}^{n-1}C_r + {}^{n-2}C_r + {}^{n-2}C_{r+1}$

$= {}^{n-1}C_r + {}^{n-2}C_r + {}^{n-3}C_r + {}^{n-3}C_{r+1}$

$= {}^{n-1}C_r + {}^{n-2}C_r + {}^{n-3}C_r + \ldots\ldots + {}^{r+1}C_{r+1}$

$= {}^{n-1}C_r + {}^{n-2}C_r + {}^{n-3}C_r + \ldots + \dfrac{r+1}{r+1}\,{}^rC_r =$ L.H.S.

Sol.11.(a) we will calculate

$101^{50} - 99^{50} = (100 + 1)^{50} - (100 - 1)^{50}$

$= (100^{50} + {}^{50}C_1 \cdot 100^{49} + {}^{50}C_2 \cdot 100^{48} + \ldots\ldots + 1)$

$= (100^{50} - {}^{50}C_1 \cdot 100^{49} + {}^{50}C_2 \cdot 100^{48} - \ldots\ldots + 1)$

$\Rightarrow \quad 101^{50} - 99^{50} = 2\Big[{}^{50}C_1 \cdot 100^{49}$

$+ {}^{50}C_3 100^{47} + \ldots + {}^{50}C_{49} \cdot 100\Big]$

$\Rightarrow \quad 101^{50} - 99^{50} = 100^{50} + 2 \cdot {}^{50}C_3 \times 100^{47}$

$+ \ldots\ldots + 2\,{}^{50}C_{49} \cdot 100$

$\Rightarrow \quad 101^{50} - 99^{50} = 100^{50} +$ Positive integer

$\Rightarrow \quad 101^{50} = 99^{50} + 100^{50} +$ Positive Integer

$\Rightarrow \quad 101^{50} > 99^{50} + 100^{50}$

(b)

L.H.S. $= {}^{2n-2}C_{n-2} + {}^{2n-2}C_{n-1} + {}^{2n-2}C_{n-1} + {}^{2n-2}C_n$

$= {}^{2n-1}C_{n-1} + {}^{2n-1}C_n = {}^{2n}C_n > \dfrac{4n}{n+1}$

Sol.12 Find term not containing x is $\left(1 + x + \dfrac{7}{x}\right)^{11}$

$\frac{7}{x}$	x	1	
0	0	0	${}^{11}C_0\, 7^0 \cdot {}^{11}C_0\, 1^0 \cdot {}^{11}C_{11}\, 1^{11} = 1$
1	1	9	${}^{11}C_1\, 7^1 \cdot {}^{10}C_1\, 1^1 \cdot 1^9 =$
2	2	7	${}^{11}C_2\, 7^2 \cdot {}^{9}C_2 \cdot 1 =$
3	3	5	${}^{11}C_3\, 7^3 \cdot {}^{8}C_3 =$
4	4	3	${}^{11}C_4\, 7^4 \cdot {}^{7}C_3 =$
5	5	1	${}^{11}C_5\, 7^5 \cdot {}^{6}C_5 =$

$= 1 + \sum_{k=1}^{5} {}^{11}C_k\, 7^k \cdot {}^{11-k}C_k$

or

1	x	1		
1	0	0	${}^{11}C_{11}\, 1^{11}$	$= 1$
9	1	1	${}^{11}C_9\, {}^2C_1\, 7^1$	$= {}^{11}C_2\, {}^2C_1\, 7^1$
7	2	2	${}^{11}C_7\, {}^4C_2\, 7^2$	$= {}^{11}C_4\, {}^4C_2\, 7^2$
5	3	3	${}^{11}C_5\, {}^6C_3\, 7^3$	$= {}^{11}C_6\, {}^6C_3\, 7^3$
3	4	4	${}^{11}C_3\, {}^8C_4\, 7^4$	$= {}^{11}C_8\, {}^8C_4\, 7^4$
1	5	5	${}^{11}C_1\, {}^{10}C_5\, 7^5$	$= {}^{11}C_{10}\, {}^{10}C_5\, 7^5$

$= 1 + \sum_{k=1}^{5} {}^{11}C_{2k} \cdot {}^{2k}C_k\, 7^k$

Sol.13 For coeff. of x^5

degrees in of x		$(1 + x^2)^5$		$(1 + x)^4$
5	=	0	+	Not Possible
5	=	2	+	3
5	=	4	+	1

$\Rightarrow$ coeff of x^2 in $(1+ x^2)^5$ & coeff. of x^3 in $(1 + x)^4$

or coeff of x^4 in $(1 + x^2)^5$ & coeff of x in $(1 + x)^4$

$= {}^5C_1\,(x^2)^1 \cdot {}^4C_3 x^3 + {}^5C_2\,(x^2)^2 \cdot {}^4C_1\, x^1$

$= (5.4 + 10.4)\, x^5$

coeff. of $x^5 = (20 + 40) = 60$

Sol.14 (i) $(1 + x + x^2 + x^3)^{11} = [(1 + x) + x^2 (1 + x)]^{11}$

$= (1 + x)^{11} (1 + x^2)^{11}$

For coeff. of x^4

$(1 + x)^{11}$	$(1 + x^2)^{11}$
4	0
2	2
0	4

$= \quad x^4$ in $(1 + x)^{11} + \{x^2$ in $(1 + x)^{11}\}$

$\{x^2$ in $(1 + x^2)^{11}\} + x^4$ in $(1 + x^2)^{11}$

$= {}^{11}C_4 + {}^{11}C_2 \cdot {}^{11}C_1 + {}^{11}C_2 = {}^{11}C_4 + {}^{11}C_2 \cdot 12$

$= \dfrac{11 \cdot 10 \cdot 9 \cdot 8}{4 \cdot 3 \cdot 2} + \dfrac{12 \cdot 11 \cdot 10}{2} = 330 + 660 = 990$

(ii) ${}^6C_0\,(2 - x)^6 + {}^6C_1\,(2 - x)^5\,(3x^2)^1$

$+ {}^6C_2\,(2 - x)^4\,(3x^2)^2 +$ above 4 degrees

term of $x^4 =$

${}^6C_0\, {}^6C_4\, 2^2 \cdot (-x)^4 + {}^6C_1\, {}^5C_2\, 2^3\, (-x)^2\, 3 \cdot x^2$

$+ {}^6C_2 \cdot {}^4C_0\, 2^4 \cdot 9 \cdot x^4$

$= (15 \cdot 4 + 6 \cdot 10 \cdot 24 + 15 \cdot 16 \cdot 9)\, x^4$

$= (60 + 1440 + 2160)\, x^4 = 3660\, x^4$

$\therefore$ coeff. of x^4 is 3660

Sol.15 **(i)** $(2 + 3x)^9$ when $x = \frac{3}{2}$

$$\Rightarrow \frac{9+1}{\left|\frac{2}{3x}\right|+1} - 1 \leqslant r \leqslant \frac{9+1}{\left|\frac{2}{3x}\right|+1}$$

$$\Rightarrow \frac{10}{\frac{4}{9}+1} - 1 \leqslant r \leqslant \frac{10}{\frac{4}{9}+1} \Rightarrow \frac{77}{13} \leqslant r \leqslant \frac{90}{13}$$

$\Rightarrow\ 5.9 \leqslant r \leqslant 6.9 \Rightarrow r = 6 \qquad (\because r \in N)$

$\therefore T_7 = T_{6+1}$

$$= {}^9C_6 (2)^3 \left(3 \cdot \frac{3}{2}\right)^6 = \frac{9 \cdot 8 \cdot 7}{3 \cdot 2} \cdot \frac{2^3 \cdot 3^{12}}{2^6} = \frac{3^{13} \cdot 7}{2}$$

(ii) $(3 - 5x)^{15}$ when $x = \frac{1}{5}$

$$\Rightarrow \frac{16}{\left|\frac{3}{-5x}\right|+1} - 1 \le r \le \frac{16}{\left|\frac{3}{-5x}\right|+1}$$

$\Rightarrow 3 \leqslant r \leqslant 4 \Rightarrow r = 3$ or $r = 4$

$\therefore T_4 = {}^{15}C_3\, 3^{12} \left(-5 \cdot \frac{1}{5}\right)^3$

$$= -\,{}^{15}C_3\, 3^{12} = -\frac{15 \cdot 14 \cdot 13}{3 \cdot 2 \cdot 1} 3^{12} = -455 \times 3^{12}$$

$T_5 = {}^{15}C_4\, 3^{11} (-1)^4 = {}^{15}C_4\, 3^{11}$

$$= \frac{15 \cdot 14 \cdot 13 \cdot 12}{3 \cdot 2} = 455 \times 3^{12}$$

$\therefore |T_4| = |T_5| = 455 \times 3^{12}$

Sol.16 $s_n = \dfrac{1-q^{n+1}}{1-q}$ … (i)

$$S_n = \frac{1-\left(\frac{q+1}{2}\right)^{n-1}}{1-\left(\frac{q+1}{2}\right)} = \frac{\left[2^{n+1} - (q+1)^{n+1}\right]}{2^n(1-q)} \quad \text{… (ii)}$$

now L.H.S. $= \sum_{r=0}^{n} T_{r+1} = \sum_{r=0}^{n} {}^{n+1}C_{r+1}\, s_r$

$$= \sum_{r=0}^{n} {}^{n+1}C_{r+1} \left(\frac{1-q^{r+1}}{1-q}\right)$$

$$= \frac{1}{(1-q)} \left[\sum_{r=0}^{n} {}^{n+1}C_{r+1} - \sum_{r=0}^{n} {}^{n+1}C_{r+1} q^{r+1}\right]$$

$$= \frac{1}{(1-q)} \left[\left({}^{n+1}C_1 + {}^{n+1}C_2 + \ldots\ldots + {}^{n+1}C_{n+1}\right)\right.$$

$$\left. - \left({}^{n+1}C_1 q^1 + {}^{n+1}C_2 q^2 + \ldots\ldots + {}^{n+1}C_{n+1} q^{n+1}\right)\right]$$

$$= \frac{1}{(1-q)} \left[\left(2^{n+1} - {}^{n+1}C_0\right) - \left((1+q)^{n+1} - {}^{n+1}C_0\right)\right]$$

$$= \frac{1}{(1-q)} \left[2^{n+1} - (1+q)^{n+1}\right]$$

$$= \frac{1}{(1-q)} 2^n (1-q) S_n = 2^n\, S_n = \text{R.H.S. (by (iii))}$$

Sol.17 $T_{r+1} = {}^{10}C_r (-x^2)^r$, $T_{r+1} = {}^{10}C_r\, (x)^{10-r} \left(-\frac{2}{x}\right)^r$

For $x^{10} \Rightarrow\ 2r = 10 \Rightarrow r = 5$

$T_6 = \left({}^{10}C_5\right) . x^{10} (-1)^r = {}^{10}C_r (-2)^r x^{10-2r}$

For constant term $10 - 2r = 0 \Rightarrow r = 5$

$\therefore$ coeff. of $x^{10} = -{}^{10}C_5$

$T_6 = {}^{10}C_5 (-2)^5 x^0$

$\therefore$ coeff. of constant term $= {}^{10}C_5\, (-2)^5$

requird ratio $= \dfrac{-{}^{10}C_5}{-{}^{10}C_5 . 2^5} = \dfrac{1}{2^5} = \dfrac{1}{32} = 1 : 32$

Sol.18 Term independent of x in $(1 + x + 2x^3)$

$$\left(\frac{3x^2}{2} - \frac{1}{3x}\right)^9$$

$= 1. (x^0 \text{ in B}) + 1 . (x^{-1} \text{ in B}) + 2. (x^{-3} \text{ in B})$

$$T_{r+1} = {}^9C_r \left(\frac{3}{2}\right)^{9-r} x^{18-2r} \left(\frac{-1}{3}\right)^r x^{-r}$$

$$= {}^9C_2\, \frac{3^{9-2r}}{2^{9-r}} (-1)^r x^{18-3r}$$

For constant term $x^o \Rightarrow 18 - 3r = 0 \Rightarrow r = 6$

$\therefore T_7 = {}^9C_6 \dfrac{3^{-3}}{2^3} x^o$

For constant term $x^{-1} \Rightarrow 18 - 3r = -1$
$\Rightarrow 19 = 3r$ not possible
For constant term $x^{-3} \Rightarrow 18 - 3r = -3$

$\Rightarrow 21 = 3r \Rightarrow r = 7 \quad \therefore T_8 = {}^9C_7 \dfrac{3^{-5}}{2^2} (-1)^7 x^{-3}$

$= 1 . {}^9C_6 \dfrac{1}{2^3.3^3} - 2. {}^9C_7 \dfrac{1}{2^2 3^5}$

$= \dfrac{9.8.7}{3.2} . \dfrac{1}{2^3 3^3} - 2. \dfrac{9.8}{2.1} . \dfrac{1}{2^2 3^5}$

$\dfrac{9.8}{2^3.3^3}\left(\dfrac{7}{6} - \dfrac{2}{9}\right) = \dfrac{1}{3}\left(\dfrac{21-4}{18}\right) = \dfrac{17}{54}$

Sol.19 L.H.S. = $(1 + x^2)(1 + x)^n = (1 + 2x^2 + x^4)(1 + x)^n$

$= (1 + 2x^2 + x^4)(1 + nx + \dfrac{n(n-1)}{2!} x^2 +$

$\dfrac{n(n-1)(n-2)}{3!} x^3 + ...)$

$1 + nx + \left(2 + {}^nC_2\right) x^2 + (2n + {}^nC_3) x^3$ + higher

terms of x by compair with $a_0x^o + x_4x + a_2x^2 + a_3x^3 + ...$
$a_0 = 1, a_1 = n, a_2 = 2 + {}^nC_2, a_3 = 2n + {}^nC_2$
$\therefore a_1, a_2, a_3$ in AP $\Rightarrow 2a_2 = a_1 + a_3$
$\Rightarrow 4 + 2. {}^nC_2 = n + 2n + {}^nC_3$

$\Rightarrow 4 + 2 \dfrac{n(n-1)}{2} = 3n + \dfrac{n(n-1)(n-2)}{6}$

$\Rightarrow 6(n^2 - 4n + 4) = n(n-1)(n-2)$
$\Rightarrow (n-2)[n^2 - 7n + 12] = 0$
$\Rightarrow (n-2)(n-3)(n-4) = 0$
$\Rightarrow n = 2$ or 3 or 4

Sol.20 Coeff. a^{r-1}, a^r, a^{r+1}, in $(1 + a)^n$ are in A.P.
$\Rightarrow {}^nC_{r-1}, {}^nC_r, {}^nC_{r+1}$ in A.P.
$\Rightarrow 2 . {}^nC_r = {}^nC_{r-1} + {}^nC_{r+1}$

$\Rightarrow 2 \dfrac{n!}{r!(n-r)!} = \dfrac{n!}{(n-r+1)!(r-1)!}$

$+ \dfrac{n!}{(n-r+1)!(r+1)!}$

$\Rightarrow \dfrac{2}{r(n-r)} = \dfrac{1}{(n-r+1)(n-r)} + \dfrac{1}{r(r+1)}$

$\Rightarrow \dfrac{1}{n-1}\left[\dfrac{2}{r} - \dfrac{1}{n-r+1}\right] = \dfrac{1}{r(r+1)}$

$\Rightarrow \dfrac{2n - 3r + 2}{(n-r)(n-r+1)} = \dfrac{1}{(r+1)}$

$\Rightarrow n^2 - n(4r + 1) + 4r^2 - 2 = 0$

Sol.21 R.H.S. = ${}^nJ_r$

$= \dfrac{(1-x^n)(1-x^{n-1})(1-x^{n-2})....(1-x^{n-r+1})}{(1-x)(1-x^2)(1-x^3)....(1-x^r)}$

$= \dfrac{(1-x^n)(1-x^{n-1})(1-x^{n-2})...(1-x^{n-r+1})}{(1-x)(1-x^2)(1-x^3)...(1-x^r)}.$

$\dfrac{(1-x^{r+1})(1-x^{r+2})...(1-x^{n-r-1})(1-x^{n-r})}{(1-x^{r+1})(1-x^{r+2})...(1-x^{n-r-1})(1-x^{n-r})}$

$= \dfrac{(1-x^n)(1-x^{n-1})(1-x^{n-2})...(1-x^{n-r+1})}{(1-x)(1-x^2)(1-x^3)...(1-x^r)(1-x^{r+1})}.$

$\dfrac{(1-x^{n-r})(1-n^{n-r-1})...(1-x^{r+2})(1-x^{r+1})}{(1-x^{r+2})...(1-x^{n-r-1})(1-x^{n-r})}$

by given definition $= {}^nJ_{n-r}$ = L.H.S.

Aliter : $\dfrac{{}^nJ_{n-r}}{{}^nJ_r} = 1 \Rightarrow {}^nJ_{n-r} = {}^nJ_r$

Sol.22 Let $S = \sum_{k=0}^{n} {}^nC_k \sin kx . \cos(n-k)x$

${}^nC_0 \sin 0x \cos(n-0)x + {}^nC_1 \sin 1x \cos(n-1)x$
$+ .... + {}^nC_n \sin nx \cos(n-n)x$ ....(1)
arrange reverse order
$S = {}^nC_0 \sin nx \cos 0x + {}^nC_1 \sin(n-1)x \cos 1x$
$+ .... + {}^nC_n \sin 0x \cos nx$ ...(2)
adding (1) & (2) {and apply sin (A + B)}
$\Rightarrow 2S = {}^nC_0 \sin(nx + 0x) + {}^nC_1 \sin(nx - x + x)$
$+....+ {}^nC_n \sin(0x + nx)$
$\Rightarrow 2S = {}^nC_0 \sin nx + {}^nC_1 \sin nx + {}^nC_2 \sin nx +$
$.... + {}^nC_n \sin nx$
$\Rightarrow 2S = [{}^nC_0 + {}^nC_1 + {}^nC_2 + ... + {}^nC_n] \sin nx$
$\Rightarrow 2S = 2^n . \sin nx \Rightarrow S = 2^{n-1} \sin nx$ = R.H.S.

Sol.23 If $(1 + x)(1 + x + x^2)(1 + x + x^2 + x^3)$
$\ldots (1 + x + x^2 + \ldots + x^n)$
$= a_0 + a_1x + a_2x^2 + \ldots$ ....(1)

(a) max power of x in L.H.S. =
= multily all x's in each factor in L.H.S. (max. degree)
$= x^1.x^2.x^3.x^4\ldots x^n = x^{1+2+3+\ldots+n} = x^{n(n+1)/2} = x^k$

& R.H.S. $= a_0 + a_1x + a_2x^2 + \ldots + a_{n(n+1)/2}\, x^{\frac{n(n+1)}{2}}$

No. of terms are $= \frac{n(n+1)}{2} + 1 = \frac{n^2+n+2}{2}$

(b) Replace $x \to \frac{1}{x}$ in equation (1)

$$\left(1+\frac{1}{x}\right)\left(1+\frac{1}{x}+\frac{1}{x^2}\right)\left(1+\frac{1}{x^2}+\frac{1}{x^3}\right)$$

$$\left(1+\frac{1}{x}+\frac{1}{x^2}+\frac{1}{x^n}\right) = a_0 + \frac{a_1}{x} + \frac{a_2}{x^2} + \ldots + \frac{a_k}{x^k}$$

$$\frac{(1+x)}{x} \cdot \frac{(1+x+x^2)}{x^2}\ \frac{(1+x+x^2+x^3)}{x^3} \ldots$$

$$\frac{(1+x+x^2+\ldots+x^n)}{x^n} = \frac{a_0x^k + a_1x^{k-1} + a_2x^{k-2} + \ldots + a_k}{x^k}$$
...(2)

By compoarision (1) & (2)
$a_0 = a_k \Rightarrow$ constant
$a_1 = a_{k-1} \Rightarrow$ coeff. of x
$a_2 = a_{k-2} \Rightarrow$ coeff. of x^2

(c) put x = 1, by (1)
$a_0 + a_1 + a_2 + a_3 + \ldots = 2 \cdot 3 \cdot 4 \ldots n \cdot (n + 1)$
$\Rightarrow a_0 + a_1 + a_2 + a_3 + \ldots = (n + 1)!$ ....(3)
Put x = –1, by (1)
$a_0 - a_1 + a_2 - a_3 + \ldots$
$= (1 - 1)(1 - 1 + 1)(1 - 1 + 1 - 1)$
$\Rightarrow a_0 - a_1 + a_2 - a_3 + \ldots = 0$ ....(4)
Add (3) & (4)
$2[a_0 + a_2 + a_4 + \ldots] = (n + 1)!$

$a_0 + a_2 + a_4 + \ldots = \frac{(n+1)!}{2}$ ....(5)

Substract (3) & (4)
$2[a_1 + a_3 + a_5 + \ldots] = (n + 1)!$

$a_1 + a_3 + a_5 + \ldots = \frac{(n+1)!}{2}$ ....(6)

From (5) & (6) H.P.

Sol.24 **(a)** x^6 in $(ax^2 + bc + c)^9$

ax^2	bx	c			
0	6	3	$^9C_0a^0.^9C_6b^6.c^3$	=	$84b^6c^3$
1	4	4	$^9C_1a^1.^8C_4b^4.c^4$	=	$630b^4c^4a^1$
2	2	5	$^9C_2a^2.^7C_2b^2.c^5$	=	$756a^2b^2c^5$
3	0	6	$^9C_3a^3.^6C_0b^0c^6$	=	$84a^3c^6$

$= 84b^6c^3 + 630 \cdot ab^4c^4 + 756\, a^2b^2c^5 + 84\, a^3c^6$

(b) $x^2y^3z^4$ in $(ax - by + cz)^9$
$= {}^9C_2a^2 \cdot {}^7C_3(-b)^3 \cdot {}^4C_4c^4$
$= -36 \cdot 35\, a^2b^3c^4 = -1260\, a^2b^3c^4$

(c) $a^2b^3c^4d$ in $(a - b - c + d)^{10}$
$= {}^{10}C_2(1)^2\, {}^8C_3(-1)^3 . {}^5C_4(-1)^4 \cdot {}^1C_1(1)^1$
$= -45 \cdot 56 \cdot 5.1 = -2520 \cdot 5 = -12600$

Sol.25 Let $x - 3 = y \Rightarrow (x - 2) = (x - 3) + 1 = y + 1$

$$\Rightarrow \sum_{r=0}^{2n} a_r(1+y)^r = \sum_{r=0}^{2n} b_r y^r$$

$\Rightarrow a_0 + a_1(1 + y) + a_2(1 + y)^2 + \ldots$
$+ a_{n-1}(1 + y)^{n-1} + a_n(1 + y)^n + a_{n+1}(1 + y)^{n+1} + \ldots$
$+ a_{2n}(1 + y)^{2n}$
$= b_0 + b_1y + b_2y^2 + \ldots + b_ny^n + \ldots + b_{2n}y^{2n}$
Compair of the coeff. of y^n both side.
In L.H.S. coeff. of y^n
${}^nC_n$ in $(1 + y)^n$
${}^{n+1}C_n$ in $(1 + y)^{n+1}$
$\vdots \quad \vdots$
${}^{2n}C_n$ in $(1 + y)^{2n}$
Coeff. of y^n in L.H.S. = Coeff. of y^n in R.H.S.
${}^nC_n + {}^{n+1}C_n + {}^{n+2}C_n + \ldots + {}^{2n}C_n = b_n$

$$\left\{{}^nC_n = \frac{(n+1)}{(n+1)}\,{}^nC_n = {}^{n+1}C_{n+1}\right\}$$

$$\because \underbrace{{}^{n+1}C_{n+1} + {}^{n+1}C_n} + {}^{n+2}C_n + \ldots + {}^{2n}C_n = b_n$$

$$\Rightarrow \underbrace{{}^{n+2}C_{n+1} + {}^{n+2}C_n} + \ldots + {}^{2n}C_n = b_n$$

$\Rightarrow {}^{n+3}C_{n+1} + {}^{2n}C_n = b_n$
Similarly
$\Rightarrow {}^{2n}C_{n+1} + {}^{2n}C_n = b_n \Rightarrow {}^{2n+1}C_{n+1} = b_n$ H.P.

Sol.26 Let (x + 3) = a & (x + 2) = b

Now $a^{n-1} + a^{n-2} b + a^{n-3} b^2 + ... + b^{n-1}$

$$= a^{n-1}\left[1+\left(\frac{b}{a}\right)+\left(\frac{b}{a}\right)^2+\dots+\left(\frac{b}{a}\right)^{n-1}\right]$$

$$= a^{n-1}\frac{\left(1-\left(\frac{b}{a}\right)^n\right)}{\left(1-\frac{b}{a}\right)} = a^{n-1}\frac{(a^n-b^n)}{a^n}\times\frac{a}{a-b}$$

$$= \frac{(x+3)^n-(x+2)^n}{(x+3)-(x+2)} = (x+3)^n-(x+2)^n$$

= Coeff. of x^r in $(x + 3)^n$ – coeff. of x^r in $(x + 2)^n$

$= {}^nC_r\, 3^{n-r} - {}^nC_r\, 2^{n-r} = {}^nC_r\,(3^{n-r} - 2^{n-r})$

Sol.27 **(a)** If we finding coeff. of 9^{th} term i.e. x = 1

$$\Rightarrow \left|\frac{T_9}{T_8}\right| > 1 \ \&\ \left|\frac{T_9}{T_{10}}\right| > 1$$

$$\Rightarrow \frac{n+1}{\left|\frac{x}{5}.\frac{5}{2}\right|+1} - 1 < r < \frac{n+1}{\left|\frac{x}{5}.\frac{5}{2}\right|+1}$$

$$\Rightarrow \frac{2(n+1)}{3} - 1 < (9-1) < \frac{2(n+1)}{3}$$

$$\Rightarrow \frac{2n+2-3}{3} < 8 \quad \text{or} \quad 8 < \frac{2n+2}{3}$$

$\Rightarrow 2n - 1 < 24$ or $24 < 2n + 2$

$\Rightarrow n < \frac{25}{2}$ or $11 < n$

$\Rightarrow 11 < n < 12.5 \Rightarrow n = 12$

(b) $\left|\frac{T_4}{T_3}\right| > 1 \ \&\ \left|\frac{T_4}{T_5}\right| > 1$

$$\Rightarrow \frac{11}{\left|\frac{5}{3x}\right|+1} - 1 < 4 - 1 < \frac{11}{\left|\frac{5}{3x}\right|+1}$$

$$\Rightarrow \frac{11.|3x|}{5+3|x|} - 1 < 3 < \frac{11.|3x|}{5+|3x|}$$

$$\Rightarrow \frac{33|x|-5-3|x|}{5+3|x|} < 3 \text{ and } 3 < \frac{33|x|}{5+3|x|}$$

$\Rightarrow 30|x| - 5 < 15 + 9|x|$ and $5+9|x| < 33|x|$

$\Rightarrow 21|x| < 20$ and $15 < 24|x|$

$\Rightarrow |x| < \frac{20}{21}$ & $\frac{15}{24} < |x|$

$$\Rightarrow x \in \left(-\frac{20}{21},\frac{20}{21}\right) \text{ and } x \in \left(-\infty,-\frac{5}{8}\right) \cup \left(\frac{5}{8},\infty\right)$$

$$\Rightarrow x \in \left(-\frac{20}{21},\frac{-5}{8}\right) \cup \left(\frac{5}{8},\frac{20}{21}\right)$$

For Positive $x \in \left(\frac{5}{8},\frac{20}{21}\right)$

Sol.28 ${}^{72}C_{36} - 1 = {}^{73}C_{36} - {}^{72}C_{35} - 1$

$\{\Rightarrow {}^{72}C_{36} + {}^{72}C_{35} = {}^{73}C_{36} \Rightarrow {}^nC_r = {}^{n+1}C_r - {}^nC_{r-1}\}$

$= {}^{73}C_{36} - ({}^{73}C_{35} - {}^{72}C_{34}) - 1$

$= {}^{73}C_{36} - {}^{73}C_{35} + ({}^{73}C_{34} - {}^{72}C_{33}) - 1$

$= {}^{73}C_{36} - {}^{73}C_{35} + {}^{73}C_{34} - {}^{73}C_{33} + ...$
$+ {}^{73}C_2 - {}^{73}C_1 + {}^{73}C_0 - 1$

$$= 73\left[\frac{72!}{36!37!} - \frac{72!}{35!38!} + \frac{72!}{34!39!} .... + \frac{72!}{2!7!} - \frac{72!}{1!72!}\right]$$

= Which is divisible by 73.

Sol.29 **(a)** $N = {}^{2000}C_1 + 2 . {}^{2000}C_2 + 3. {}^{2000}C_3 + ... + 2000.{}^{2000}C_{2000}$

$N = 0.{}^{2000}C_0 + 1 . {}^{2000}C_1 + ... + 2000 . {}^{2000}C_{2000}$ ...(1)

$N = 2000\ {}^{2000}C_0 + 1999 \cdot {}^{2000}C_1 + ... + 0 \cdot {}^{2000}C_{2000}$ ...(2)

adding (1) & (2)

$\Rightarrow 2N = 2000 . [{}^{2000}C_0 + {}^{2000}C_1 + ... + {}^{2000}C_{2000}]$

$\Rightarrow 2N = 2000 . 2^{2000} = 2^3 . 5^3 .2^{2000} = 2^{2003} . 5^3$

$\Rightarrow N = 2^{2003} . 5^3$

divisors are 1, $2^1, 2^2, 2^3, \ldots, 2^{2003}, 5, 5^2, 5^3$

$5.2, 5.2^2, 5.2^3, \ldots, 5.2^{2003}$ &

$5^2.2, 5^2.2^2, 5^2.2^3, \ldots, 5^2.2^{2003}$ &

$5^3.2.5^3.2^2.5^3.2^3, \ldots, 5^3.2^{2003}$

$= 1 + 2003 + 3 + 2003 + 2003 + 2003 = 8016$

(b) $x^{2001} + \left(\frac{1}{2} - x\right)^{2001} = 0$

$\Rightarrow x^{2001} + {}^{2001}C_0\left(\frac{1}{2}\right)^{2001}(-x)^0$

$+ {}^{2001}C_1\left(\frac{1}{2}\right)^{2000}(-x)^1 + \ldots +$

${}^{2001}C_{2000}\left(\frac{1}{2}\right)^1(-x)^{2000} + {}^{2001}C_{2001}\left(\frac{1}{2}\right)^0(-x)^{2001} = 0$

$\Rightarrow {}^{2001}C_0\left(\frac{1}{2}\right)^{2001} - {}^{2001}C_1\left(\frac{1}{2}\right)^{2000} x + \ldots$

$- {}^{2001}C_{1999}\left(\frac{1}{2}\right)^2 x^{1999} + {}^{2001}C_{2000}\left(\frac{1}{2}\right)^1 x^{2000} = 0$

$\Rightarrow A_1 x^{2000} + A_2 x^{1999} + A_3 x^{1998} + \ldots$
$+ A_{2000} x + A_{2001} = 0$

$A_1 = {}^{2001}C_{2000}\left(\frac{1}{2}\right)$ & $A_2 = -\,{}^{2001}C_{1999}\left(\frac{1}{2}\right)^2$

Sum of roots $= -\frac{A_2}{A_1} = \dfrac{\left(\frac{1}{2}\right)^2 {}^{2001}C_{1999}}{\left(\frac{1}{2}\right){}^{2001}C_{2000}}$

$= \frac{1}{2}\frac{{}^{2000}C_2}{{}^{2000}C_1} = \frac{1}{2}\,\frac{(2001-1)}{2} = \frac{2000}{4} = 500$

Sol.30(a) (i) Let $(5 + 2\sqrt{6})^n = I + f$ where $0 < f < 1$

& $(5 - 2\sqrt{6})^n = f'$ where $0 < f' < 1$ $\{\because\ 5 > 2\sqrt{6}\}$

$\Rightarrow$ $I + f + f'$ = Even integer $\Rightarrow f + f' = 1$

$\because$ $0 < f + f' < 2$

$\Rightarrow$ I = even integer – $(f + f')$

= even integer – 1 = odd integer

(ii) Let $(8 + 3\sqrt{7})^n = I + f$ where $0 < f < 1$

& $(8 - 3\sqrt{7})^n = f'$ where $0 < f' < 1$ $\{\because\ 8 > 3\sqrt{7}\}$

$\Rightarrow$ $I + f + f'$ = Even integer $\Rightarrow f + f' = 1$

$\because$ $0 < f + f' < 2$

$\Rightarrow$ I = even integer – $(f + f')$

= even integer – 1 = odd integer

(iii) Let $(6 + \sqrt{35})^n = I + f$ where $0 < f < 1$

& $(6 - \sqrt{35})^n = f'$ where $0 < f' < 1$ $\{\because\ 6 > \sqrt{35}\}$

$\Rightarrow$ $I + f + f'$ = Even integer $\Rightarrow f + f' = 1$

$\because$ $0 < f + f' < 2$

$\Rightarrow$ I = even integer – $(f + f')$

= even integer – 1 = odd integer

(b)(i) Let $(3\sqrt{3} + 5)^{2n+1} = I + f$; $0 < f < 1$

& $(3\sqrt{3} - 5)^{2n+1} = f'$; $0 < f' < 1\{\therefore\ 3\sqrt{3} > 5\}$

$\Rightarrow$ $I + f - f'$ = even integer

$\Rightarrow$ $0 < f < 1$ & $0 < f' < 1$

$\therefore$ $-1 < -f' < 0 \Rightarrow -1 < f - f' < 1$

$\Rightarrow$ $f - f' = 0$ $\because$ $\{f - f'\} < 1$

$\therefore$ $f - f'$ should be integer

$\Rightarrow$ $I + 0$ = even integer $\Rightarrow I$ = even integer

(ii) Let $(5\sqrt{5} + 11)^{2n+1} = I + f$; $0 < f < 1$

& $(5\sqrt{5} - 11)^{2n+1} = f'$; $0 < f' < 1\{\therefore 5\sqrt{5} > 11\}$

$\Rightarrow$ $I + f - f'$ = even integer

$\Rightarrow$ $0 < f < 1$ & $0 < f' < 1$

$\therefore$ $-1 < -f' < 0 \Rightarrow -1 < f - f' < 1$

$\Rightarrow$ $f - f' = 0$ $\because$ $\{f - f'\} < 1$

$\therefore$ $f - f'$ should be integer

$\Rightarrow$ $I + 0$ = even integer $\Rightarrow I$ = even integer

Sol.31 Given $(7+4\sqrt{3})^n = P + \beta$ where $n, p \in N$ and $0 < \beta < 1$

Let $(7-4\sqrt{3})^n = b'$ $\therefore$ $0 < b' < 1$ $\because$ $7 > 4\sqrt{3}$

$p + \beta + b'$ = even integer

$\beta + b'$ = integer $\{\because\ 0 < \beta < 1,\ 0 < b' < 1\}$

$\therefore$ $0 < \beta + b' < 2 \Rightarrow \beta + b' = 1 \Rightarrow b' = (1-\beta)$

Now $(1-\beta)(p+\beta) = b'(p+\beta)$

$= (7-4\sqrt{3})^n (7+4\sqrt{3})^n = (49-48)^n = 1$

Sol.32 Given $(6\sqrt{6} + 14)^{2n+1} = N$, F be the fractional part of N $n \in N$

Let $(6\sqrt{6} + 14)^{2n+1} = I + F$, $0 < F < 1$

& $(6\sqrt{6} - 14)^{2n+1} = F'$, $0 < F' < 1\{\because 6\sqrt{6} > 14\}$

$\Rightarrow$ $I + F - F'$ = even integer

$\Rightarrow N - F'$ = even integer $\therefore$ $NF = NF'$

$= (6\sqrt{6} + 14)^{2n+1} . (6\sqrt{6} - 14)^{2n+1}$

$= ((6\sqrt{6})^2 - 14^2)^{2n+1} = (216-196)^{2n+1} = 20^{2n+1}$

Sol.33 Integer next above $(\sqrt{3} + 1)^{2n}$ contains 2^{n+1} as factor $n \in w$ i.e. $[\sqrt{3} + 1]^{2n} + 1$ contains 2^{n+1} as factor.

Let $(\sqrt{3} + 1)^{2n} = I + f$ $\qquad 0 < f < 1$

& $(\sqrt{3} - 1)^{2n} = f'$ $\qquad 0 < f' < 1$

$\Rightarrow I + f + f' = I + 1$ = even integer

$= (\sqrt{3} + 1)^{2n} + (\sqrt{3} - 1)^{2n}$

$= \{(\sqrt{3} + 1)^2\}^n + \{(\sqrt{3} - 1)^2\}^n$

$= \{4 + 2\sqrt{3}\}^n + \{4 - 2\sqrt{3}\}^n$

$= 2^n [(2 + \sqrt{3})^n + (2 - \sqrt{3})^n]$

$= 2^n . 2[{}^nC_0 2^m + {}^nC_2 2^{m-2} (\sqrt{3})^2 + ...]$

$= 2^{n+1}$ [any positive integer]

which is divisible by 2^{n+1}

Sol.34 Given $(3 + \sqrt{5})^n = I + F$ $\therefore$ $0 < F < 1$, $n \in N$

Let $(3 - \sqrt{5})^n = F'$, $0 < F' < 1$ $(\because\ 3 > \sqrt{5})$

$\Rightarrow I + F + F'$ = even integer $\Rightarrow$ $F + F' = 1$

Now $I + F = \rho + \sigma$ .... (i) & $F' = \rho - \sigma$ ....(ii)

$I + (F + F') = 2\rho \Rightarrow \rho = \left(\frac{I+1}{2}\right)$

Put in (ii) $\sigma = \frac{1}{2}(I + 2F - 1)$ $\qquad$ H.P.

Sol.35 $\frac{{}^{2n}C_n}{n+1}$ is an integer $\forall\ n \in N$

$= \frac{{}^{2n}C_n}{(n+1)}[(2n+2) - (2n+1)]$

$= {}^{2n}C_n \left[\frac{2(n+1)}{(n+1)} - \frac{(2n+1)}{(n+1)}\right]$

$= 2.\ {}^{2n}C_n - \frac{(2n+1)}{(n+1)}\ {}^{2n}C_n$ $\qquad \left\{{}^{n+1}C_{r+1} = \frac{(n+1)}{(r+1)}\ {}^nC_r\right\}$

$= 2.\ {}^{2n}C_n - {}^{2n+1}C_{n+1}$

${}^{2n}C_n$ & ${}^{2n+1}C_{n+1}$ are integer

& subtraction of two integer is also integer.

Aliter :

$\frac{{}^{2n}C_n}{n+1}(n + 1 - n) = {}^{2n}C_n - \frac{n}{n+1}\ {}^{2n}C_n$

$= {}^{2n}C_n - \frac{n}{n+1}\frac{(2n)!}{n!\ n!} = {}^{2n}C_n - \frac{2n!}{(n+1)!(n-1)!}$

$= {}^{2n}C_n - {}^{2n}C_{n+1}$ or ${}^{2n}C_n - {}^{2n}C_{n-1}$